EE3054 - Homework 5 - Due Oct. 7 2005

1. Find the inverse z-transforms of the following:
   (a) \( \frac{z}{z^2} \) with ROC \( 2 < |z| < \infty \)
   (b) \( \frac{z}{z-2} \) with ROC \( 0 < |z| < 2 \)
   (c) \( \frac{3}{1-2.5z^{-1}+z^{-2}} \) with ROC \( 0.5 < |z| < 2 \).

2. Mr. Rich opens a savings bank account on January 1, 2005 with an initial deposit of $10000. The interest rate is 1%/month computed based on the amount of money in the account on the fifteenth day of the month. Mr. Rich withdraws $500 from the bank account on the last day of each month. How much money is in the bank account on January 1, 2006? Is this a BIBO stable system?

3. A numerical method to approximately solve differential equations is to discretize them by approximating \( \frac{dy}{dt} \) with a finite difference, i.e.,

   \[
   \frac{dy(nT_s)}{dt} \approx \frac{y(nT_s) - y((n-1)T_s)}{T_s} \tag{1}
   \]

   where \( T_s \) is the sampling period. \( y(nT_s) \) which is the value of the continuous-time signal \( y(t) \) at time \( nT_s \) forms the \( n^{th} \) sample of the associated discrete-time signal \( y[n] \triangleq y(nT_s) \).

   Given the differential equation

   \[
   \frac{d}{dt} y = -y + x \tag{2}
   \]

   with initial condition \( y(0) = 1 \) and input signal \( x(t) = t \), we want to find the value of the output signal \( y(t) \) at time \( t = 10 \) seconds. Do this via the following steps.

   (a) Pick the sampling period to be \( T_s = 0.01 \) seconds. Show that finding \( y(t) \) at \( t = 10 \) seconds is equivalent to finding \( y[n] \) at \( n = 10 \ast 100 \).

   (b) Find the associated discrete-time difference equation using (1).

   (c) Show that the given initial condition and input signal are equivalent to \( y[0] = 1 \) and \( x[n] = 0.01n \). Remember that, by definition, \( x[n] \triangleq x(nT_s) \).

   (d) Solve the difference equation found in part (b) with initial condition \( y[0] = 1 \) and the input signal \( x[n] = 0.01n \). Find \( y[n] \) at \( n = 10 \ast 100 \). This value is equal to the value of \( y(t) \) at time \( t = 10 \) seconds.