1. (10 points) Consider the parallel interconnection of the two causal systems, System 1 and System 2, shown below.

![System Diagram]

The impulse response of System 1 is given to be \( h_1[n] = 3^{-n}u[n] \) and the transfer function of System 2 is given to be \( H_2(z) = \frac{1}{z-1} \).

(a) Is the overall system LTI? Is the overall system FIR or IIR? Briefly explain your reasoning.

(b) Find the transfer function of the overall system. Find the poles of the overall system.

(c) Find the impulse response of the overall system.

2. (12 points) You are given four systems with impulse responses \( h_1[n] \), \( h_2[n] \), \( h_3[n] \), and \( h_4[n] \), respectively:

\[
\begin{align*}
    h_1[n] &= \delta[n] \\
    h_2[n] &= u[-n] \\
    h_3[n] &= 2^n u[n] \\
    h_4[n] &= \delta[n+1] + \delta[n+2].
\end{align*}
\]

(a) Sketch the signals \( h_1[n], h_2[n], h_3[n], \) and \( h_4[n] \).

(b) Based on your sketches in part (a), decide which ones of the four systems are causal?

(c) Based on your sketches in part (a), decide which ones of the four systems are BIBO stable?

Briefly explain the reasoning for your answers to parts (b) and (c).

3. (12 points) You are given a system with the impulse response \((0.5)^n u[n]\).

(a) Find the difference equation of the system (i.e., the equation relating the output signal \( y \) and the input signal \( x \)).
(b) Find the frequency response $H(e^{j\omega})$ of the system.

(c) If the input signal is $x[n] = 3\cos(\frac{\pi}{4}n)u[n]$, find the steady-state output signal.

4. (8 points) Consider the signal $x[n] = \delta[n] + \delta[n+1]$. We want to verify Parseval’s theorem for this signal. Recall that Parseval’s theorem says that the energy in the time-domain signal is equal to the energy in the frequency-domain DTFT. In mathematical terms, Parseval’s theorem says that if the DTFT of $x[n]$ is $X(e^{j\omega})$, then

$$\sum_{n=-\infty}^{\infty} x[n]x^*[n] = \frac{1}{2\pi} \int_{0}^{2\pi} X(e^{j\omega})X^*(e^{j\omega})d\omega.$$ 

Verify Parseval’s theorem for the given input signal $x[n]$ using the following procedure:

(a) Find the DTFT $X(e^{j\omega})$ of the given signal $x[n]$.
(b) Find $\sum_{n=-\infty}^{\infty} x[n]x^*[n]$.
(Hint: The easy way to do this is by first sketching the signal $x[n]x^*[n]$.)
(c) Find $\frac{1}{2\pi} \int_{0}^{2\pi} X(e^{j\omega})X^*(e^{j\omega})d\omega$ using the DTFT $X(e^{j\omega})$ that you found above in part (a).
(d) Check that your answers to parts (b) and (c) are equal.

5. (8 points + 6 Bonus Points) Consider a causal system with the difference equation

$$y[n] = -1.2y[n-1] - 0.36y[n-2] + x[n].$$

(a) Find the transfer function $H(z)$ of the system. Find the region of convergence (ROC) of $H(z)$.
(b) By the method of guessing exponentials, it can be shown that the impulse response of the system is of the form

$$h[n] = c_1\alpha_1^n u[n] + c_2n^k\alpha_2^n u[n].$$

Find the constants $\alpha_1$, $\alpha_2$, and $k$.
(c) (For extra credit - 6 Bonus Points) Find the constants $c_1$ and $c_2$ in the impulse response.