1. **(10 points)** For each of the following statements, indicate whether it is true or false and give a short (not more than two lines) reason:

(a) If an LTI system is BIBO stable, then its impulse response must be of finite length (i.e., is nonzero only for a finite interval of time).

(b) Assume that two given signals $x_1(t)$ and $x_2(t)$ have magnitude bounds $M_1$ and $M_2$, respectively, i.e., $|x_1(t)| \leq M_1$ and $|x_2(t)| \leq M_2$. Then, their convolution $x_1(t) \ast x_2(t)$ must have a magnitude bound $M_1 M_2$, i.e., it must be true that $|x_1(t) \ast x_2(t)| \leq M_1 M_2$.

(c) If a causal LTI system has a transfer function of the form $\frac{1}{s^2 + as + b}$ with $a \geq 0$ and $b \geq 0$, then it should be BIBO stable.

(d) If $x(t)$ is any causal signal, then $x(t) \delta(t+1) + x(t) \ast u(t) - \int_{-3}^{t} x(\tau)d\tau = 0$.

(e) If the Laplace transform of a signal $x(t)$ is $X(s)$, then the Laplace transform of the signal $\int_{-\infty}^{t} x(\tau)e^{j\omega_0 \tau}d\tau$ is $\frac{1}{s}X(s - j\omega_0)$.

2. **(14 points)** Consider a ball of mass $M$ tied to an elastic string (like a yo-yo). Let $y(t)$ be the length of the string at time $t$. Due to friction, the ball experiences a force $D\dot{y}(t)$. The elasticity of the string causes a force $K y(t)$. The force due to gravity is $Mg$. Therefore, the differential equation of this system is

$$M\ddot{y}(t) = Mg - D\dot{y}(t) - Ky(t).$$

Consider the string length $y(t)$ to be the system output and the gravitational force $x(t) = Mg$ to be the system input.

(a) Find the transfer function of the system.

(b) Using the one-sided Laplace transform, find $y(t)$ for $t \geq 0$ if the string is initially not stretched (i.e., $y(0) = 0$) and the ball is initially at rest (i.e., $\dot{y}(0) = 0$). Assume $M = 1$ kg, $D = 7$ kg/s, and $K = 12$ kg/s$^2$. (Hint: Remember that since $x(t)$ is a constant, we have $X(s) = \frac{Mg}{s}$).

(c) Physically, we expect that after a lot of time, any oscillations should die out and the ball should come to rest at a position at which the elastic pull of the string should exactly cancel out the gravitational force. Check whether this happens in the solution that you found in part (b). Explain why or why not in the context of BIBO stability of the system.

(d) Instead of the initial conditions in part (b), assume now that the string was initially not stretched (i.e., $y(0) = 0$), but that the ball was initially falling at $1$ m/s (i.e., $\dot{y}(0) = 1$). Find $y(t)$ for $t \geq 0$ in this case. What happens to $y(t)$ as $t \rightarrow \infty$?

3. **(10 points)** You are given a causal LTI system with the transfer function $\frac{s}{s+3}$.

(a) Find the impulse response of the system. (Hint: $\frac{s}{s+3} = 1 - \frac{3}{s+3}$).
(b) Find the step response of the system.
(c) Using convolution, find the output signal of the system if the input signal is $x(t) = \sin(t)u(t)$. Assume zero initial conditions.
(d) With the same input signal as in part (c), find the steady-state output signal.

4. (8 points) You are given four systems $A$, $B$, $C$, and $D$ with the transfer functions $H_A(s)$, $H_B(s)$, $H_C(s)$, and $H_D(s)$ shown below:

\[
H_A(s) = \frac{1}{s} \\
H_B(s) = \frac{1}{s+1} \\
H_C(s) = \frac{1}{s-1} \\
H_D(s) = \frac{1}{s^2 + 1}.
\]

The impulse responses of the four systems are sketched in the figures below (but not in the same order). Match each system ($A$, $B$, $C$, $D$) to the corresponding impulse response (one of $h_1(t), h_2(t), h_3(t), h_4(t)$).

5. (8 points + 6 Bonus Points) This question is also based on the four systems $A$, $B$, $C$, and $D$ given in Question 4.

(a) For each of the systems $A$, $B$, $C$, and $D$, indicate if it is causal or noncausal.
(b) For each of the systems $A$, $B$, $C$, and $D$, indicate if it is BIBO stable or not.
(c) (For extra credit - 6 Additional Points) For this subquestion, disregard the impulse responses shown in the figures above and assume that the four systems $A$, $B$, $C$, and $D$ are all causal. Find the impulse response of the cascade interconnection of the systems $A$, $B$, $C$, and $D$. Also, find the step response of the cascade interconnection of the systems $B$, $C$, and $D$. 