1. (8 points) Consider the system
\[
y[n] = y[n-1] + y[n-2] + x[n].
\]
(a) Is this system LTI? Is this system FIR or IIR? Briefly justify your answers.
(b) Find the transfer function \( H(z) \) of the system. Find the poles of the system.

2. (10 points) Consider the cascade interconnection of Systems 1 and 2 shown below.

The impulse response of System 1 is given to be \( h_1[n] = 2^n u[n] \) and the difference equation of System 2 is given to be \( y[n] = y_1[n] + y_1[n-1] \) where, as shown in the figure, \( y_1[n] \) is the input signal for System 2.
(a) Find the transfer function of the overall system.
(b) Find the impulse response of the overall system.

3. (12 points) Given a function \( H(e^{j\omega}) = \tan(\omega) \), we want to compute its inverse DTFT \( h[n] \). Find \( h[n] \) using the following steps:
(a) Show that \( \tan(\omega) = \frac{e^{j\omega} - e^{-j\omega}}{j(e^{j\omega} + e^{-j\omega})} \). Hence, show that \( H(z) = \frac{z-z^{-1}}{j(z+z^{-1})} \).
(b) Show that \( H(z) \) can be rewritten as
\[
H(z) = \frac{1}{j} \left[ -1 + \frac{1}{1 + jz^{-1}} + \frac{1}{1 - jz^{-1}} \right].
\]
(c) Show that

\[ h[n] = \frac{1}{j}(-\delta[n] + (\frac{j}{n}) u[n] + (j)^n u[n]). \]

4. (12 points) You are given a system with difference equation

\[ y[n] = 0.5y[n-1] + x[n]. \]

(a) Find the frequency response \( H(e^{j\omega}) \) of the system. Find the magnitude response \( |H(e^{j\omega})| \) and the phase response \( \angle H(e^{j\omega}) \).

(b) If the input signal is \( x[n] = 2 \sin(\frac{n}{3} + \frac{\pi}{3}) u[n] \), find the steady-state output signal.

5. (8 points) The step response of an unknown system is observed to be \( (\delta[n] + \delta[n - 1] + \delta[n - 2]) \). Find the impulse response of the system \( (\text{Hint: the easy way to do this is by plotting } u[n] \text{ and } u[n-1]) \). Also, find the equation relating the input signal \( x[n] \) and the output signal \( y[n] \) of the system.