Homework

Pole placement and observer design

1. The linearized dynamics of an inverted pendulum are as follows:

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 5 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -2 \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}
\end{align*}
\] (1)

(a) Using state feedback, place the closed loop poles at -1, -2 and -1 ± j.

(b) Design a full-order observer with poles at -3 ± 2j, -4 and -5.

(c) Draw a block diagram realization of the whole system (i.e. plant, observer and controller).

2. Consider the undamped harmonic oscillator

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -w_0^2 x_1 + u(t) \\
y &= x_2
\end{align*}
\] (2)
(a) Design an observer/state feedback compensator to control the position $x_1(t)$. Place the state-feedback controller controller poles at $s = -w_0 \pm j$ and both observer poles at $s = -3w_0$.

(b) Draw block diagram realization of the whole system.

3. Consider the dynamics of a DC motor

$$L \frac{di}{dt} + Ri = V - V_b$$

$$J \ddot{\theta} + B \dot{\theta} = \tau - \tau_l$$  \hspace{1cm} (3)

Ignoring $L$ and $\tau_l$ and knowing that $V_b = K_b \dot{\theta}$ and $\tau = K_m i$, we can rewrite the motor dynamics as follows:

$$J \ddot{\theta} + (B + \frac{K_b K_m}{R}) \dot{\theta} = \frac{K_m}{R} i$$  \hspace{1cm} (4)
Define $x_1 = \theta, x_2 = \dot{\theta}, u = \frac{K_m}{R}i$. Then,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{J}(B + \frac{K_b K_m}{R}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$  \hspace{1cm} (5)

Let $y = \theta = x_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Take $-\frac{1}{J}(B + \frac{K_b K_m}{R}) = -10$.

(a) Design a full-order observer for $x_2$. Draw a block diagram of the overall closed-loop system with a state feedback. Place the poles of the system at -1 and -12 and the observer poles at -20.

(b) Simulate the overall system.