EL6253 Homework

1. Consider the following system:

\[
\dot{x} = \begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix} x + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u.
\]

Rewrite the above dynamics in the controllable canonical form through an appropriate change of coordinates. Also, design a control law to place the poles of the closed-loop system at \(-1, -2, -3\). Write the control law both in terms of the transformed state representation \(z\) and in terms of the original state representation \(x\).

2. Consider the following system:

\[
\dot{x} = \begin{bmatrix}
0 & 1 & 2 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix} x + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u \\
y = [1, 0, 1] x.
\]

(a) Rewrite the above dynamics in the controllable canonical form through an appropriate change of coordinates.

(b) Design a control law to place the poles of the closed-loop system at \(-1\) and \(-3 \pm 2j\). Write the control law both in terms of the transformed state representation \(z\) and in terms of the original state representation \(x\).

(c) Design a full-order observer to estimate the state \(x\) using the measurement of the output \(y\); place the observer poles at \(-2\) and \(-1 \pm j\).