Given a linear system of form

\[
\begin{align*}
\dot{x} & = Ax + Bu \\
y & = Cx,
\end{align*}
\]

(1)
a functional observer to estimate a linear functional \( w = Kx \) is given by

\[
\begin{align*}
\dot{z} & = Fz + \overline{G}_1 u + \overline{G}_2 y \\
\hat{w} & = Ly + Qz
\end{align*}
\]

(2)
where \( F \) is a stable matrix and the following equations hold:

\[
\begin{align*}
PA - FP & = \overline{G}_2 C \\
\overline{G}_1 & = PB \\
K & = QP + LC
\end{align*}
\]

(3)
where \( P \) is some matrix.

Defining \( e = \hat{w} - Kx \), we get \( e = Qz - QPx = Q(z - Px) \).

Defining \( e_z = z - Px \),

\[
\begin{align*}
\dot{e}_z & = Fz + \overline{G}_1 u + \overline{G}_2 y - P(Ax + Bu) \\
& = Fz - FPx \\
& = F(z - Px) \\
& = Fe_z.
\end{align*}
\]

(4)
Hence, \( e_z \) goes to zero asymptotically as \( t \to \infty \). Since \( e = Qe_z \), we see that \( e = \hat{w} - Kx \) also goes to zero asymptotically. Therefore, \( \hat{w} \) provides an asymptotically converging estimate of \( Kx \).