1. Which of the following functions are positive definite functions (i.e., \( f(0) \) is zero and \( f(x) \) is positive for any non-zero \( x \))? Justify.
   
   (a) \( f(x) = x^2 + x^4 \) for \( x \in \mathbb{R} \)
   
   (b) \( f(x) = x^4 e^x \) for \( x \in \mathbb{R} \)
   
   (c) \( f(x) = (x^T P x)^2 \) for \( x \in \mathbb{R}^2 \) and \( P = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \).

2. Consider the system

\[
\begin{align*}
\dot{x}_1 &= -x_1 - x_1^3 + 2x_2 \\
\dot{x}_2 &= -2x_2 - x_1.
\end{align*}
\]

Using the Lyapunov function \( V = \frac{1}{2}(x_1^2 + x_2^2) \), what can you say about the stability properties of this system?

3. Consider the system

\[
\dot{x} = -x^3 + x^5
\]

(a) Find the equilibrium point(s) of this system.

(b) Is this system globally asymptotically stable to the origin (the point \( x = 0 \)), i.e., do trajectories starting from any initial value of \( x \) converge to the origin? Justify.

(c) Is this system locally asymptotically stable to the origin, i.e., do trajectories starting from initial values of \( x \) in some small neighborhood of the origin converge to the origin? Justify.

(d) Using \( V = \frac{1}{2}x^2 \) and noting for what values of \( x \) would \( \dot{V} \) be negative, estimate the region of attraction of the system to the origin.